Quantum states with a positive partial transpose are useful for metrology

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The relation between the various subsets of entangled states have been studied for a long time. It has been conjectured by A. Peres that no bound entangled state violates a Bell inequality [4], which, after numerous attempts, has been recently refuted [5]. The search for counterexamples has been hindered by the fact that the conjecture is very close to be true.

In this work we ask the question: are there bound entangled states that are metrologically useful? Can states that do not violate any Bell inequality be metrologically useful? Finding such states numerically seems to be as easy as finding a needle in the haystack, since we need to *maximize* a convex function over a convex set. There have been results concerning entanglement criteria with several quantum Fisher information terms detecting PPT entangled states as well as concerning the metrological usefulness of multipartite states that are not PPT with respect to all bipartions [3, 1, 2]. However, it is a famously hard open problem of quantum information theory whether states with only PPT entanglement can be useful for metrology [2].

We give an affirmative answer to the question above. We show that there are bound entangled states that outperform all separable states metrologically, as depicted in Fig. 1. Below, we summarize the four main contributions of this paper.

(i) We present multiqubit quantum states that are metrologically useful, while having a positive partial transpose with respect to all bipartitions. In this way, we make sure that the metrological advantage compared to separable states cannot be attributed to the non-PPT bipartitions.

(ii) We also present several bipartite PPT states for dimensions from 3×3 to 12×12 that outperform separable states in quantum metrology. The metrological advantage of these states compared to separable states is very robust to noise. Thus, such states might be realized in experiments with photons or trapped cold ions.

(iii) We show an iterative method based on semidefinite programming (SDP) that can generate such states very efficiently. The method, starting from a given initial state, provides a series of PPT quantum states with a rapidly increasing metrological usefulness.

(iv) We now turn to the relation of metrological usefulness and other convex sets of quantum states. We show that quantum states with a local hidden variable model, i.e., not violating any Bell inequality can be metrologically useful. We present such states with a positive as well as with a non-positive partial transpose [2].

We present an iterative method based on semidefinite programming to find bound entangled states that are useful for metrology, which converges very fast as shown in Fig. 2.

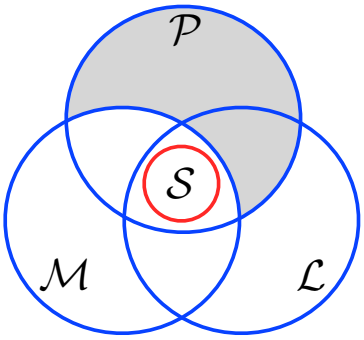

Figure 1: Various convex sets of quantum states represented by circles: ($\mathcal{P}$) PPT states, ($\mathcal{M}$) states that are not useful for metrology, ($\mathcal{S}$) separable states, ($\mathcal{L}$) states with a local hidden variable model. (grey area) Metrologically useful PPT states. Such states are in $\mathcal{P}\backslash\mathcal{M}$, where "$\backslash$" denotes the difference between two sets.

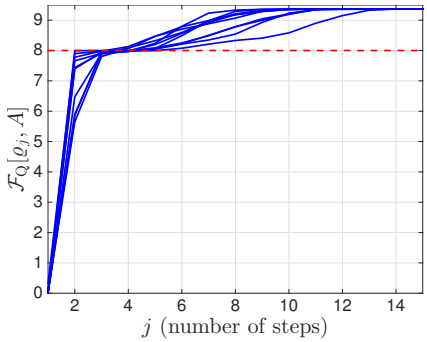

Figure 2: Convergence to the optimal quantum Fisher information during the generation of the 4×4 bound entangled state. (solid) 10 attempts are shown. After 15 steps, the algorithm converged to the optimal value. (dashed) The maximal value of the quantum Fisher information for separable states.

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