Structure of the resource theory of quantum coherence

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Introduction. Quantum resource theories provide a strong framework for studying fundamental properties of quantum systems and their applications for quantum technology. The basis of any quantum resource theory is the definition of *free states* and *free operations*. Free states are quantum states which can be prepared at no additional cost, while free operations capture those physical transformations which can be implemented without consumption of resources. Having identified these two main features, one can study the basic properties of the corresponding theory, such as possibility of state conversion, resource distillation, and quantification. An important example is the resource theory of entanglement, where free states are separable states, and free operations are local operations and classical communication [1].

In the resource theory of *quantum coherence* [2, 3], free states are identified as incoherent states $\rho = \sum_i p_i |i\rangle\langle i|$, i.e., states which are diagonal in a fixed specified basis $\{|i\rangle\}$. The choice of this basis depends on the particular problem under study, and in many relevant scenarios such a basis is naturally singled out by the unavoidable decoherence. The definition of free operations within the theory of coherence is not unique, and several approaches have been discussed in the literature, based on different physical (or mathematical) considerations [3]. An important framework is known as *incoherent operations* (IO) [2]. The characterizing feature of IO is the fact that they admit an incoherent Kraus decomposition, i.e., they can be written as $\Lambda(\rho) = \sum_j K_j \rho K_j^\dagger$, where each of the Kraus operators K_j cannot create coherence individually, $K_j |m\rangle \sim |n\rangle$ for suitable integers n and m. This approach is motivated by the fact that any quantum operation can be interpreted as a selective measurement in which outcome j occurs with probability $p_j = \mathrm{Tr}[K_j \rho K_j^\dagger]$, and the state after the measurement is given by $K_j \rho K_j^\dagger / p_j$. An IO can then be interpreted as a measurement which cannot create coherence even if one applies post-selection on the measurement outcomes.

Having identified the relevant class of free operations, it is important to ask about a description of those operation efficient in the physical dimension. Such an efficient description seems crucial for a rigorous investigation of the corresponding resource theory. In entanglement theory, it is known that the set of local operations and classical communication is notoriously difficult to capture mathematically. Here, we address this question for the resource theory of coherence, focusing on the class IO. We show that this class admit a minimal standard form, and use these results to give a full solution for the mixed-state conversion problem of a single qubit. We further introduce the set of Gibbs-preserving strictly incoherent operations, and solve the mixed-state conversion problem for a single qubit also for these operations [4].

Results. A general quantum operation, acting on a Hilbert space of dimension d, admits a decomposition with at most d^2 Kraus operators – this is the maximum *Kraus rank*. However, since incoherent Kraus operators have a very specific structure, it is unclear if this result also transfers to IO. As we have shown in ref. [4], any incoherent operation acting on a Hilbert space of dimension d admits a decomposition with at most $d^4 + 1$ incoherent Kraus operators. For $d = 2$ and $d = 3$, this number can be improved to 5 and 39 respectively. This result assists in a rigorous investigation of the resource theory of coherence, since they significantly reduce the number of free parameters for the set IO. To demonstrate the power of these results, we provide a full characterization for all possible state transformations via single-qubit IO in Fig. 1 [4]. As a second application for our results, we introduce and investigate Gibbs-preserving strictly incoherent operations, thus relating the resource theories of quantum coherence and quantum thermodynamics. Also in this case we fully solve the mixed-state conversion problem for a single qubit [4].

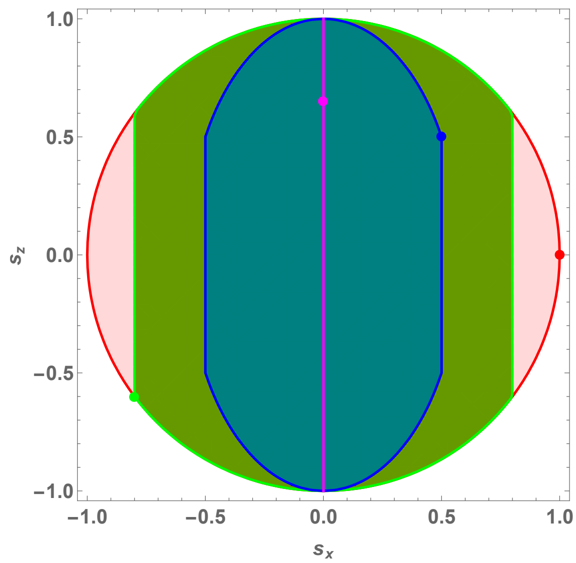

Figure 1: Achievable region for IO on a single-qubit.

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