Quantum phase transitions and correlation detection in the measurement space

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Macroscopic features of N-qubit systems can be described by discrete distributions $\widetilde{Q}\left(m,n,k\right)$, $0\leq m,n,k\leq N$, in a 3-dim space of symmetric measurements [1]. Such distributions contain full and non-redundant information about any collective (invariant under particle permutations) observable and are very useful for visualization and analysis of general properties of arbitrary states of large quantum systems. In the macroscopic limit, $N\gg 1$, $\widetilde{Q}$-functions tend to smooth distributions and are *bounded* by Gaussian envelopes.

We will show that the localization (extension in the measurements space) and the gaussianity (that quantifies how well a $\widetilde{Q}$-distribution is described by its Gaussian envelope) of the $\widetilde{Q}$-function can be used for characterization of long-range correlations and detection of quantum phase transitions. We analyze all-order phase transitions in Hamiltonians $H=H(\mathbf{g})$, where $\mathbf{g}=(g_1,g_2,...)$ are some coupling constants, and show that the derivative of the Hellinger distance between an "initial" $\widetilde{Q}_{\mathbf{g}_0}$ and "current" $\widetilde{Q}_{\mathbf{g}}$ distributions, $\partial_{\mathbf{g}}D\left(\mathbf{g},\mathbf{g}_0\right)$, being

$$D\left(\mathbf{g},\mathbf{g}_0\right)=1-\frac{1}{2^N}\sum_{\mathbf{h}}\sqrt{\widetilde{Q}_{\mathbf{g}}\left(\mathbf{h}\right)\widetilde{Q}_{\mathbf{g}_0}\left(\mathbf{h}\right)},\ \mathbf{h}=(m,k,n),$$

detects the critical values of the coupling constants, while the analytical properties of the $\widetilde{Q}$-functions allows to determine the type of correlations in stable (under changing of the parameters $\mathbf{g}$) regions. We analyze both ground states and thermal states of N-qubit Hamiltinians.

For instance, for the Ising model

$$H=-\sum_i\left[\sigma_i^x\sigma_{i+1}^x+g\sigma_i^z\right], \tag{1}$$

the derivative $\partial_g D\left(g,g_0\right)$ indicates that the phase-transition occur at $g=1$ (see Fig.1); the $\widetilde{Q}$-distributions for $g<1$ and $g>1$ have de-localized and localized forms correspondingly (see Figs.2.3) We apply our method to different Hamiltonians for analysis both phase-transitions as, e.g.

$$H=\sum_i\left[\sigma_z^i\sigma_z^{i+1}-g_1\sigma_x^i+g_2\sigma_z^i\sigma_x^{i+1}\sigma_z^{i+2}\right],$$

and percolation effects, as e.g.

$$H=\sum_{i,j}g_{ij}\sigma_z^i\sigma_z^j.$$

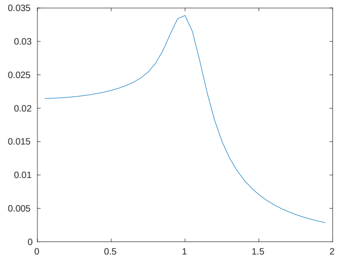

Figure 1: Derivative of the Hellinger distance $\partial_g D\left(g,g_0\right)$ for the Ising Hamiltonian (1) has a sharp maximum at the critical value $g=1$; $T=0.1$, $N=14$.

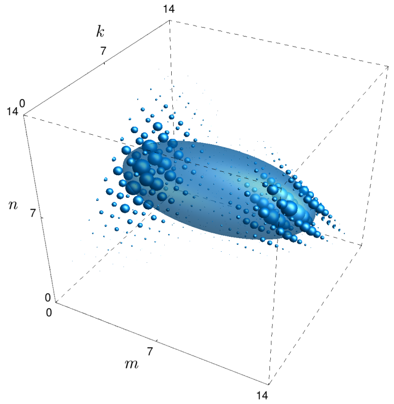

Figure 2: $\widetilde{Q}$-function (and its Gaussian envelope) for the Hamiltonian (1) at $g=0$ (and up to $g\approx 0.9$).

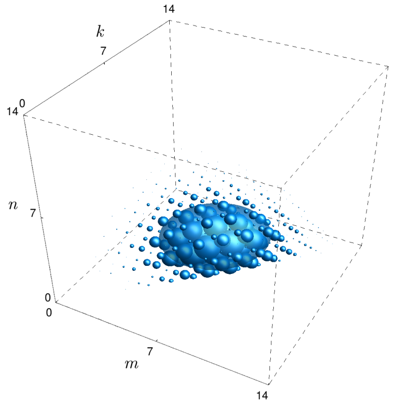

Figure 3: $\widetilde{Q}$-function (and its Gaussian envelope) for the Hamiltonian (1) at $g=2$ (and from $g\approx 1.1$).

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[1] A.B. Klimov, C. Munoz Phys. Rev.A **70**, 062101 (2014); M. Gaeta, C. Munoz, A.B. Klimov Phys. Rev.A **93**, 062107 (2016); M. Gaeta, C. Munoz, A.B. Klimov J. Phys. A **51** (2018)