Golden Ratio entanglement in hexagonally poled nonlinear crystals

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Two-dimensional nonlinear photonic crystals [1] offer a high degree of flexibility for engineering the classical and quantum properties of parametric processes,because of the presence of multiple vectors of the nonlinear lattice providing quasi phase matching. We analyse here the quantum state of twin photons and twin beams generated by parametric-down conversion (PDC) in a hexagonally poled $\chi^{(2)}$ crystal (Fig.1a). This photonic crystal is characterized by the coexistence of two nonlinear processes[2], seeded by the two fundamental vectors of the reciprocal lattice of the nonlinearity (Fig.1b). In the spectral-angular domain of the fluorescence light there are special points where phase matching occurs simultaneously for both processes, and where down-converted photons may originate by either processes. In the high-gain regime the two possibilities add coherently and stimulate each other, giving rise to unusual isolated hot spots in the parametric emission.

In the quantum domain, we describe a general scenario of three-mode entanglement and we show that the photonic crystal is equivalent to a PDC process followed by a beam-splitter, acting thus as a compact nonlinear interferometer. By properly tilting the pump beam, we identify a particular condition where two triplets of hot spots coalesce into four entangled modes (Fig 1c). We call this condition *super-resonance* because all the interacting waves have the same spatial transverse modulation as the nonlinear lattice. Here, in the high-gain regime, the brightness of hot-spots is further enhanced, and we demonstrate that the rate of growth of parametric light along the crystal increases by the famous *Golden Ratio* of the segment , $\phi = \frac{1+\sqrt{5}}{2}$. The 4-mode interaction at superresonance can be approximately described by the following set of parametric equations

$$\begin{cases} \frac{\partial}{\partial z}\hat{a}_0 &= g\left[\hat{b}_0^\dagger + \hat{b}_2^\dagger\right] \\ \frac{\partial}{\partial z}\hat{a}_2 &= g\,\hat{b}_0^\dagger \\ \frac{\partial}{\partial z}\hat{b}_0 &= g\left[\hat{a}_0^\dagger + \hat{a}_2^\dagger\right] \\ \frac{\partial}{\partial z}\hat{b}_2 &= g\,\hat{a}_0^\dagger \end{cases} \tag{1}$$

where $\hat{a}_{0,2}$ and $\hat{b}_{0,2}$ are signal and idler photon annihilation operators for the modes shown in Figs.1c, d, and g is proportional to the pump amplitude and to the $\chi^{(2)}$ nonlinearity. We then show that the dynamics described by Eq.(1) is equivalent to i) two independent parametric processes, with unbalanced gains $g\phi$ and $\frac{g}{\phi}$, ii) followed by an unbalanced beam-splitter that mixes the two processes according to the Golden Ratio. We offer a physical picture of the occurrence of such particular number, based on the microscopic processes taking place.

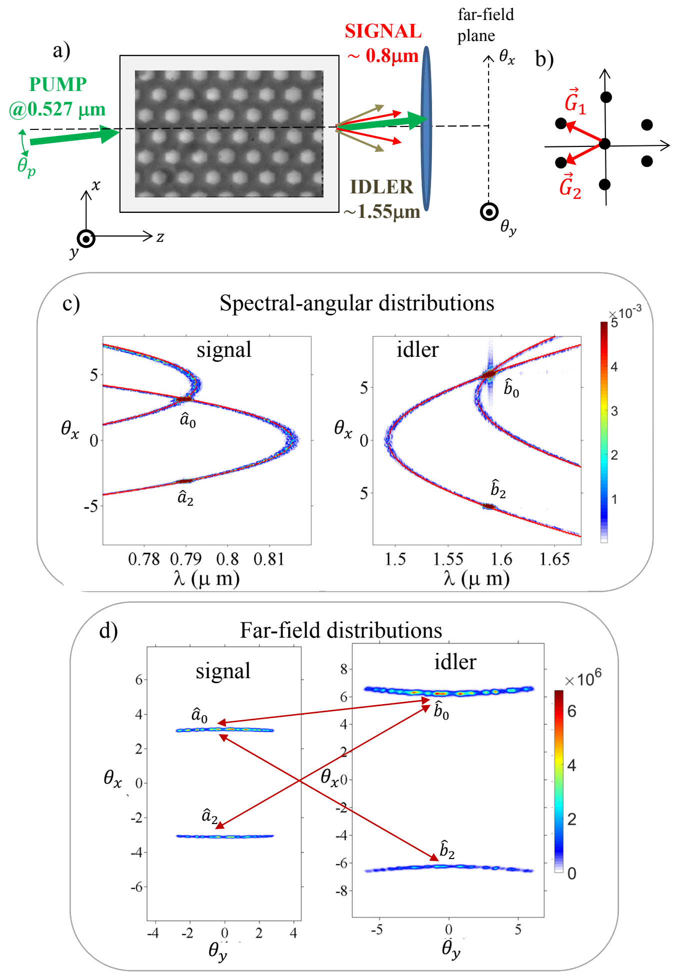

Figure 1: a) Parametric-down-conversion in a hexagonally poled photonic crystal. b) Vectors of the reciprocal lattice. c) and d): 3D +1 stochastic simulations, showing: c) The spectral- angular distribution of the parametric light, at super-resonance, with the 4 interacting modes of Eq.(1), corresponding to as many hot-spots. The vertical scale is truncated at 0.3% of the peak value, in order to reveal the underlying standard fluorescence, distributed around the phase-matching curves (red lines in the plot). d) Corresponding far-field distributions, where hot-spots emitted at different wavelengths form approximately straight lines.

[1] V. Berger, Phys. Rev. Lett. **81**, 4136 (1998).

[2] K. Gallo et al., Appl. Phys. Lett. 98, 161113 (2011); M. Levenius et al. Appl.Phys. Lett. 101, 121114 (2012)