Incommensurate Crystals of Trapped Ions in Optical Cavities

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In this work we report on the optomechanical dynamics of ion chains, whose vibrations couple with the high-Q mode of an optical cavity. The dynamics results from the interplay between the long-range Coulomb repulsion and the cavity-induced interactions. The latter are due to multiple scatterings of laser photons inside the cavity and become relevant when the laser pump is sufficiently strong to overcome photon decay. We study the stationary states of ions coupled with a mode of a standing-wave cavity as a function of the cavity and laser parameters, when the typical length scales of the two self-organizing processes, Coulomb crystallization, leading to an ion crystal with lattice constant a in the center, and photon-mediated interactions, inducing a potential with periodicity b, are almost commensurate as displayed in Fig. 1 a).

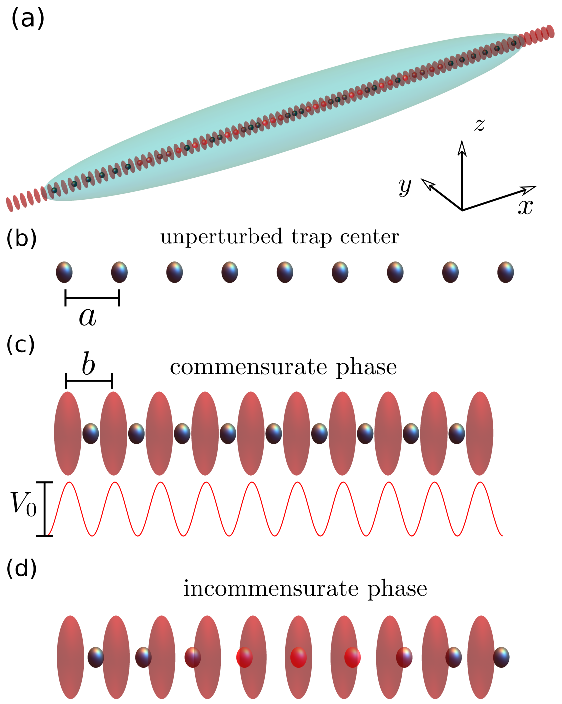

Figure 1: (a) A chain of trapped ions (dark and red dots) interacting with an optical lattice (indicated by the periodic light intensity modulations in red) can undergo a commensurate-incommensurate transition by tuning the interparticle distance at the center of the harmonic trap. The red ions indicate the kinks. Subplot (b) shows the chain at the trap center and in absence of the lattice. Subplot (c) illustrate the commensurate phase, where the lattice periodicity b with lattice strength V_0 matches the unperturbed interparticle distance a. When a is slightly varied such that a/b is an incommensurate ratio the phase becomes incommensurate and kinks forms. Subplot (d) illustrates one kink.

In the regime of low cooperativities, cavity backaction can be neglected. By tuning the ratio of a/b we expect this model to simulate the ground state of a one-dimensional crystal growing on top of a substrate with periodicity b. If the substrate potential is sufficiently weak, and the mismatch between a and b is small, the ions adapt to the lattice and ground state is commensurate as shown in Fig. 1 c). For larger values of the mismatch, the ground state becomes incommensurate with localized density distortions called kinks arising in the chain as in Fig 1 d). We derive a field theory for the kinks and analyze their properties. Using a mean-field approach, we then determine the phase diagram in the presence of the long-range Coulomb interactions and of the external trapping potential. We analyze the spectrum of small oscillations and show that it is strongly modified by the presence of the kinks. In the incommensurate phase, in fact a new branch of low-frequency modes appears that is associated with the correlated motion of kinks.

We then study the case of strong cavity nonlinearities, where the creation of kinks can significantly change the intracavity field through the dynamical Stark shift. In the past, it has been shown that cavity backaction modifies the sliding to pinned transition for a/b chosen as the golden mean, where bistable parameter regions can arise and superlubric and stick-slip dynamics may coexist [1]. It has also been demonstrated that the cavity can act as a thermal reservoir, that can cool the chain vibrations to temperatures that depend on the cavity parameters [2]. We analyze the effects of infinite ranged cavity induced interactions on the commensurate-incommensurate transition and show that the transition becomes of first order which is associated with bistable behavior of the cavity field. This bistability of the cavity field is accompanied by a bistability of the kinks, where now multiple profiles of the kinks are supported. Finally, we show that the transition can be observed in chains of dozens of ions in a harmonic trap at finite temperatures and identify the salient properties at the transition. In particular, we show that the state of the chain is imprinted in the radiation emitted at the cavity output.

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[1] T. Fogarty, C. Cormick, H. Landa, V. M. Stojanović, E. Demler, and G. Morigi, Phys. Rev. Lett. **115**, 233602 (2015).

[2] T. Fogarty, H. Landa, C. Cormick, and G. Morigi, Phys. Rev. A **94**, 023844 (2016).