Superposition principle for classical probabilities in quantum suprematism representation

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In quantum suprematism representation of spin-$1/2$ states [1] the density matrices of these states are expressed [2] in terms of three probability distributions of three dihotomic random variables. These probability distributions give the probabilities $0 \leq p_1, p_2, p_3 \leq 1$ of spin $-1/2$ projections $m = +1/2$ on three perpendicular directions $\vec{x}, \vec{y}, \vec{z}$. The probabilities satisfy for pure states the equality $\sum_{j=1}^{3}(p_j - 1/2)^2 = 1/4$. It means that known parametrization of qubit states by vectors in Bloch ball is mapped onto vector parameters given by the probabilities with components belonging to the probability ball with radius equal $1/2$. The points in the ball were bijectively mapped onto the points on the plane. These points on the plane belong to triada of Malevich's squares which illustrates the spin-$1/2$ states in representation called quantum suprematism picture. The quantum correlations in qubit states provide inequality for area of the Malevich's squares. In [3] the superposition principle of quantum states was formulated in terms of density operators. It means that one can add two density matrices of pure quantum states and obtain the density matrix of the pure superposition states. Since all the states are expressed in terms of the probabilities the superposition principle of spin-$1/2$ states can be expressed as the rule for addition of the probabilities determining two states $|\psi_1\rangle$ and $|\psi_2\rangle$ which provide the state $|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$ also expressed in terms of the probabilities. In quantum suprematism representation it means that for two triadas of Malevich's squares determining the states $\hat{\rho}_1 = |\psi_1\rangle\langle\psi_1|$, $\hat{\rho}_2 = |\psi_2\rangle\langle\psi_2|$ the triada of the superposition state $\hat{\rho}_\psi = |\psi\rangle\langle\psi|$ is given in explicit form. To get the result we express the components of vector $|\chi\rangle = (\chi_1, \chi_2)$ in terms of the probabilities. They read $\chi_1 = \sqrt{p_3}$, $\chi_2 = \sqrt{p_3(1-p_3)}e^{-i\phi}$, where $\cos\phi = (p_1 - 1/2)/\sqrt{p_3(1-p_3)}$. Using this expression and probabilities $\lambda_3 = |c_1|^2$, λ_1 and λ_2 determining qubit state $|\psi_0\rangle = \left(|c_1|, \sqrt{|c_1|^2(1-|c_1|^2)}e^{-i\phi}\right)$, where $\cos\phi = (\lambda_1 - 1/2)/\sqrt{|c_1|^2(1-|c_1|^2)}$, $\sin\phi = (\lambda_2 - 1/2)/\sqrt{|c_1|^2(1-|c_1|^2)}$ we obtain the nonlinear addition rule for the two qubit states.

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